

Fundamental plasmid strings and black rings

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ABSTRACT: We construct excited states of fundamental strings that admit a semiclassical description as rotating circular loops of string. We identify them with the supergravity solutions for rotating dipole rings. The identification involves a precise match of the mass, radius and angular momentum of the two systems. Moreover, the degeneracy of the string state reproduces the parametric dependence of the entropy in the supergravity description. When the solutions possess two macroscopic angular momenta, they are better described as toroidal configurations (tubular loops) instead of loops of string. We argue that the decay of the string state can be interpreted as superradiant emission of quanta from the ergoregion of the rotating ring.

KEYWORDS: Black Holes, Superstrings and Heterotic Strings.

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1. Introduction

Fundamental strings contain excited states that admit a semiclassical description. These can be regarded as macroscopic strings, and are of interest for many reasons. On the one hand, since they are built using perturbative string theory, many of their properties can be understood in detail. On the other hand, being macroscopic objects, they can source spacetime fields and give rise to supergravity solutions of string theory at low energy. Such dual descriptions, first advanced in [1–5], are at the basis of many recent developments in the microphysics of black holes, e.g., [6], and novel AdS/CFT dualities [7–9]. Semiclassical string states also provide a handle on certain solvable regimes of the gauge/string correspondence [10]. Additionally, they may play a role in cosmology, see e.g., [11].

In this paper we are mainly concerned with the use of semiclassical string states for the microscopic interpretation of black hole-like objects in supergravity. Specifically, we make a connection between a class of states of fundamental strings, namely circular loops of string, following [12, 13], and the supergravity solutions that describe black rings with a dipole of the Kalb-Ramond field [14, 15]. These loops of fundamental string do not possess any conserved gauge charges, and they are not supersymmetric. They also possess angular momentum: the loop is rotating, and its centrifugal repulsion prevents its collapse.

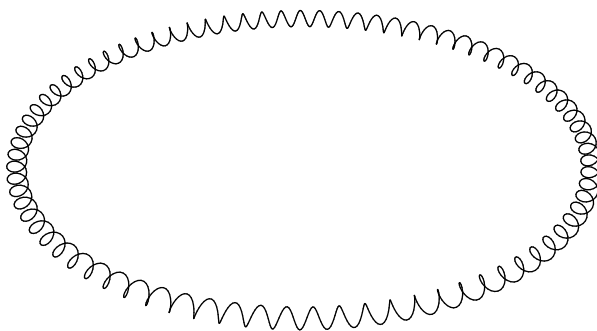


Figure 1: Plasmid string, describing a rotating loop of string. The rotation of the ring corresponds to the helical advance of the string. For a generic rotating string loop, the profile of the string oscillations is not a circular helix but a wiggly structure.

The rigid rotation of a fundamental string is not possible because of reparametrization invariance of the worldsheet, but a loop of excited fundamental string can rotate. An example involving fermionic excitations was presented in [12]; here we will discuss a simpler construction using bosons. The essence of the idea is captured pictorially by the notion of a *plasmid* string (figure 1):¹ a helical string that closes in on itself on a circle, with the helical advance of the string resulting in the coherent rotation of the state along the ring circle. For a generic state the oscillations of the string do not have the profile of a circular helix, but are replaced by small-scale wiggles of the string whose propagation along the circular loop give the ring its angular momentum. They also give rise to a large degeneracy of the macroscopic loop.

Our identification of the microscopic string state and the supergravity solution is accurate when the ring is *thin*, in the sense that its radius is large and its self-gravitation is weak. In this regime we are able to precisely match the relations between the mass, angular momentum, radius and dipole charge (i.e., the winding n_w around the ring circle) that appear in both sides of the equivalence.

Properly speaking, the solutions we study are not black, since they have naked singularities instead of regular horizons. However, in the spirit of [5], one can associate a stretched horizon to these solutions, and as we shall see, the area of this horizon reproduces the degeneracy of the string state up to an undetermined numerical coefficient. In fact a regular horizon is expected to arise when higher-derivative corrections to the action are included — they are then *small* black rings. The possible connection between these string states and black rings was anticipated in [14].

When the helix radius becomes macroscopically large, which happens when the angular momentum on the plane orthogonal to the plane of the ring is macroscopic, the configuration is not adequately described as a ring, but rather as a tubular loop. In fact we shall argue that it is a loop of (a certain) supertube. These configurations can be analyzed in

¹A plasmid is a circular molecule of a double-stranded DNA helix. The fundamental plasmid string is instead single-stranded.

the approximation in which the radius of the loop is very large, and they reproduce the properties of the corresponding string states.

None of these string states is supersymmetric and therefore they are expected to decay once string interactions are turned on. We shall argue for an interpretation of the decay in terms of supergravity: it corresponds to the spontaneous emission of superradiant modes from the ergoregion that surrounds the ring. The quantitative development of this idea is technically somewhat involved and is left for the future, but at least we are able to provide a qualitative description of the equivalence using the ideas in [16, 17].

The paper is divided into three main sections, each one describing a different construction of the loop of string: section 2 builds it as a classical solution to the Nambu-Goto equations; section 3 obtains it as a quantum state of string theory; section 4 then compares these configurations with the supergravity solution for a dipole ring. The last section comments on some consequences and possible extensions of these results.

2. Classical fundamental plasmid string

In this section we present solutions of the classical equations of motion for a fundamental bosonic string describing circular loops stabilized by angular momentum, i.e., plasmid strings. The lowest dimension in which our construction can be made is $D = 5$, and for clarity we will only work out explicitly this case. The extension to higher dimensions is nevertheless straightforward.

Our starting point is the Nambu-Goto action for a string in five-dimensional flat space,

$$S_{\text{NG}} = -\frac{1}{2\pi\alpha'} \int \sqrt{-\gamma} d\sigma d\tau, \tag{2.1}$$

where $1/(2\pi\alpha')$ is the tension of the string, γ denotes the determinant of the induced metric on the worldsheet, $\gamma_{mn} = \eta_{\mu\nu} \partial_m X^\mu \partial_n X^\nu$, and $m, n = 0, 1$, $\mu, \nu = 0, \dots, 4$ are the worldsheet and spacetime indices respectively. The equations of motion obtained from this action are

$$\partial_m (\sqrt{-\gamma} \gamma^{mn} \partial_n X^\nu) = 0. \tag{2.2}$$

It is convenient to rewrite these equations in the conformal gauge where $\gamma_{mn} = \sqrt{-\gamma} \eta_{mn}$. In this gauge the equations of motion are just the two-dimensional wave equation,

$$\partial_\sigma^2 X^\mu - \partial_\tau^2 X^\mu = 0. \tag{2.3}$$

Furthermore, we can use the residual gauge freedom to fix $X^0 = p_0 \tau$, so the previous equations become

$$\partial_\sigma^2 X^i - \partial_\tau^2 X^i = 0, \tag{2.4}$$

where X^i denotes the four-dimensional spatial vector describing the position of string in our five-dimensional spacetime. The most general solution of these equations is

$$X^i = \frac{1}{2} [A^i(\tau - \sigma) + B^i(\tau + \sigma)]. \tag{2.5}$$

Finally we still have to make sure that our solution satisfies the constraints, which in our gauge impose the conditions

$$|\partial_\sigma A^i|^2 = |\partial_\sigma B^i|^2 = p_0^2. \tag{2.6}$$

Armed with the most general solution, we can now try to look for configurations of the type we are interested in, namely, stationary circular loops stabilized by angular momentum. At first, this sounds impossible to do. A fundamental bosonic string does not have any longitudinal degrees of freedom since any such apparent motion can always be compensated by a change of gauge. On the other hand, we can imagine the situation in which small-scale wiggles propagate along the string in a nearly circular loop, making it possible for it to have some angular momentum perpendicular to the ring. The travelling wiggles produce a centrifugal force that balances the tension of the string and therefore allow for stationary configurations. A family of such solutions can be easily written in the conformal gauge as

$$\begin{aligned} X^0 &= p_0 \tau, \\ X^1 &= R \cos(2n_w(\tau - \sigma)), \\ X^2 &= R \sin(2n_w(\tau - \sigma)), \\ X^3 &= \frac{R n_w}{N} \cos(2N(\tau + \sigma)), \\ X^4 &= \frac{R n_w}{N} \sin(2N(\tau + \sigma)), \end{aligned} \tag{2.7}$$

with $p_0 = 4n_w R$. Taking $0 \leq \sigma < \pi$, the solution represents a circular loop of radius R , winding n_w times on the X^1 - X^2 plane, and stabilized against collapse by a chiral excitation of N helical turns of amplitude Rn_w/N on the X^3 - X^4 plane. The winding number n_w is not topologically conserved since the string, which is dynamically stabilized by rotation and not by topology, can be continuously shrunk to zero radius. These solutions have been previously considered, for a different purpose, in [44, 45].

It is straightforward to see that these solutions indeed fulfill the constraints described above, since both $\partial_\sigma A^i$ and $\partial_\sigma B^i$ parametrize circles of radius p_0 . We have chosen these circles to lay on perpendicular planes in order to avoid the presence of cusps in these solutions. There are, of course, many other solutions similar to this one. In particular, we see that the macroscopic shape of the loop can be arbitrary since the only constraint on the function $A^i(\tau - \sigma)$ is given by eq. (2.6), which basically fixes the parametrization of A^i but not its shape. The reason for this is that the balance between the tension and the centrifugal force produced by the wiggles is a local phenomenon that ensures that at each point on the string the effective tension vanishes.² Here, however, we only focus on the circular string case.

As we anticipated, the solutions presented above have a non-vanishing 1-2 component of the angular momentum, which depends on the winding number as well as the radius of

²Solutions of this type are known to exist in chiral superconducting string models [18] where the role of the chiral wiggles is played by a neutral current on the string worldsheet and the strings states are the so-called chiral vortons [19].

the loop as

$$J_{12} = \frac{1}{2\pi\alpha'} \int_0^\pi d\sigma (X^1 \partial_\tau X^2 - X^2 \partial_\tau X^1) = \frac{n_w R^2}{\alpha'}. \quad (2.8)$$

On the other hand, we can see that N not only controls the amplitude of the small oscillations but also enters in the calculation of the 3-4 component of the angular momentum,

$$J_{34} = \frac{1}{2\pi\alpha'} \int_0^\pi d\sigma (X^3 \partial_\tau X^4 - X^4 \partial_\tau X^3) = \frac{n_w^2 R^2}{\alpha' N}. \quad (2.9)$$

For a very large value of N we find ourselves with an almost perfectly circular string loop with only J_{12} angular momentum. This is the kind of solution we want to identify with our black ring solutions in the supergravity description.³

An interesting point about this gauge is that σ parametrizes the energy along the string, so we can easily read off the total of energy, i.e., the mass, of a particular configuration from the total range of σ . In our example the mass is

$$M = \frac{1}{2\pi\alpha'} \int_0^\pi p_0 d\sigma = \frac{2Rn_w}{\alpha'}. \quad (2.10)$$

From the results above we also obtain

$$J_{12} = M^2 \frac{\alpha'}{4n_w}. \quad (2.11)$$

This is always below the Regge bound $J = M^2 \alpha'$, even when $n_w = 1$. Hence we expect the existence of many other string configurations that satisfy (2.11) for given M , J_{12} and n_w . This is, there must exist a large degeneracy of these states. It appears that the origin of this degeneracy is the possibility of varying the small-scale structure of the loop without altering the relations (2.10) and (2.11).

This is easily seen to be the case. The plasmid solution above is only one of the simplest possible configurations with these properties but we can write the most general solution for a circular ring with chiral perpendicular travelling wiggles as

$$\begin{aligned} X^0 &= 4n_w R \tau, \\ X^1 &= \frac{1}{2} A^1 (\tau - \sigma) = R \cos(2n_w (\tau - \sigma)), \\ X^2 &= \frac{1}{2} A^2 (\tau - \sigma) = R \sin(2n_w (\tau - \sigma)), \\ X^3 &= \frac{1}{2} B^3 (\tau + \sigma), \\ X^4 &= \frac{1}{2} B^4 (\tau + \sigma). \end{aligned} \quad (2.12)$$

³The case $n_w = 1$, $N = 1$ was presented in [20], but it will become clear that we do not expect it to correspond to a black ring.

The general form for the two-dimensional vector $B^i(\tau + \sigma) = B^i(\sigma_+)$ consistent with a closed string loop is given by

$$B^i(\sigma_+) = 2Rn_w \sum_{n=1}^{\infty} [c_n^i \cos(2n\sigma_+) + d_n^i \sin(2n\sigma_+)], \quad (2.13)$$

or, in terms of complex coefficients β_n^i , by

$$B^i(\sigma_+) = 2iRn_w \sum_{n \neq 0} \frac{1}{n} \beta_n^i e^{-2in\sigma_+}, \quad (2.14)$$

where

$$\beta_n^i = -\frac{in}{2}(c_n^i + id_n^i), \quad (2.15)$$

and $\beta_{-n} = \beta_n^*$. The only other constraint on these functions comes from eq. (2.6) which, in turn, imposes the following conditions on the expansion coefficients

$$\sum_{n \neq 0} \beta_n^i \beta_{m-n}^i = \delta_{m0}. \quad (2.16)$$

Therefore any solution of the form (2.12) that fulfills the requirements of eq. (2.16) represents a nearly circular loop with the same radius, mass and J_{12} as the simple plasmid string (2.7). On the other hand, these solutions have in general different values of J_{34} ,

$$J_{34} = \frac{R^2 n_w^2}{\alpha'} \sum_{n=1}^{\infty} n (c_n^3 d_n^4 - d_n^3 c_n^4) = \frac{R^2 n_w^2}{\alpha'} \sum_{n=1}^{\infty} n (\mathbf{c}_n \times \mathbf{d}_n). \quad (2.17)$$

We can now obtain, following an argument parallel to one presented in [21], an upper bound for this component of the angular momentum. Observe that

$$\begin{aligned} J_{34}^2 &= \left(\frac{R^2 n_w^2}{\alpha'} \right)^2 \sum_{m,n} n m (\mathbf{c}_n \times \mathbf{d}_n) \cdot (\mathbf{c}_m \times \mathbf{d}_m) \\ &= \left(\frac{R^2 n_w^2}{\alpha'} \right)^2 \sum_{m,n} n m [(\mathbf{c}_n \cdot \mathbf{c}_m)(\mathbf{d}_n \cdot \mathbf{d}_m) - (\mathbf{c}_n \cdot \mathbf{d}_m)(\mathbf{d}_n \cdot \mathbf{c}_m)] \\ &\leq \left(\frac{R^2 n_w^2}{\alpha'} \right)^2 \sum_{m,n} \frac{1}{4} n^2 m^2 (\mathbf{c}_n^2 + \mathbf{d}_n^2)(\mathbf{c}_m^2 + \mathbf{d}_m^2). \end{aligned} \quad (2.18)$$

Using the constraint equation (2.16) with $m = 0$, namely,

$$\sum_{n=1}^{\infty} \frac{1}{2} n^2 (\mathbf{c}_n^2 + \mathbf{d}_n^2) = 1 \quad (2.19)$$

we arrive at

$$|J_{34}| \leq \frac{R^2 n_w^2}{\alpha'}, \quad (2.20)$$

or alternatively,

$$|J_{34}| \leq n_w J_{12}. \tag{2.21}$$

Throughout the paper we take both n_w and J_{12} to be positively oriented.⁴

It is clear that the vectors \mathbf{c}_n and \mathbf{d}_n could lie in any spatial direction transverse to the 1-2 plane of the loop, and therefore the construction generalizes immediately to any $D \geq 5$. In this case we can have rotation in more independent planes.

Observe that the mass (2.10) of the plasmid string is twice the energy of a smooth static circular string wound n_w times with the same radius R . In other words, the amount of energy stored in the big loop is the same as the energy carried by the small scale structure of the string. This is, of course, not surprising, since it is this small-scale structure that acts to balance the string tension at each point along the string. In fact, we can easily derive the precise result from a simple mechanical argument. If we consider a string with n_p units of momentum wound n_w times on a circle of radius R , then its energy is well-known to be

$$E = \frac{n_p}{R} + \frac{R n_w}{\alpha'}. \tag{2.22}$$

In our construction the circle is contractible, so the string that wraps it will not be in equilibrium for all values of R but only for those that extremize the energy, i.e., those for which the ‘effective tension’

$$\frac{dE}{dR} = -\frac{n_p}{R^2} + \frac{n_w}{\alpha'} \tag{2.23}$$

vanishes. This happens when

$$\frac{n_p}{R} = \frac{R n_w}{\alpha'}, \tag{2.24}$$

which is the statement that the winding and momentum are ‘virialized’ and the total energy of the string is equally divided into them.⁵ For our circular strings the momentum along the string circle becomes rotation, so

$$n_p \rightarrow J_{12}, \tag{2.25}$$

and we see that eqs. (2.22) and (2.24) reproduce (2.10) and (2.11). Moreover, this argument shows that the circular string is at a minimum of E and so is *stable* to radial variations.

We conclude by noting that some of the states that we have constructed are ‘special’. For instance, the plasmid (2.7) overlaps itself whenever N is a multiple of $n_w > 1$. These states, however, do not decay by breaking the string since on each wind the string is parallel to itself. If N is not a multiple of n_w , but $n_w > 1$, the string will not overlap but intersect itself, possibly several times, on each turn, and this can lead to string breaking once interactions are switched on. Still, if n_w is very large the string will be almost parallel to itself at the intersections. At any rate, we need not worry much about these effects, since they will not be properties of generic string states containing many small-scale wiggles.

⁴The possibility of $n_w J_{12} < 0$ would only make a difference for heterotic strings. But the differences that appear are largely irrelevant for our purposes.

⁵For strings wound on a non-contractible circle, this corresponds to the self-T-dual compactification radius.

3. Microscopic description of a circular string loop

Our construction in the previous section was purely classical. It shows that there is a large number of string configurations that, at large scales, can be appropriately characterized as rotating loops of string. However, in order to count the degeneracy of these circular strings we must quantize the system. To this end, we turn to the full quantum description of these configurations in string theory. We begin by reviewing the procedure for defining the analogue of a coherent state description of an extended classical closed string, following the construction in [13] and generalizing it to allow for $n_w > 1$.

3.1 Fixing the gauge and solving the constraints

In light-cone gauge we set $X^+ \equiv (X^0 + X^{D-1})/\sqrt{2} = 2\alpha' p_+ \tau$. For the remaining coordinates we have the standard decomposition for closed strings into left and right-movers, which on the solutions, takes the form

$$X^i = X_L^i + X_R^i, \tag{3.1}$$

$$X_L^i = \frac{1}{2}x^i + \alpha' p^i(\tau - \sigma) + i\sqrt{\frac{\alpha'}{2}} \sum_{n \neq 0} \frac{1}{n} \alpha_n^i e^{-2in(\tau - \sigma)}, \tag{3.2}$$

$$X_R^i = \frac{1}{2}x^i + \alpha' p^i(\tau + \sigma) + i\sqrt{\frac{\alpha'}{2}} \sum_{n \neq 0} \frac{1}{n} \tilde{\alpha}_n^i e^{-2in(\tau + \sigma)}, \tag{3.3}$$

where $i = 1, \dots, D - 2$, and x^i and p^i are the center of mass position and momentum of the string loop.

After reaching light-cone gauge there is still a residual symmetry: rigid shifts of σ generated by the operator $N_L - N_R$. Following [13], we fix this residual symmetry by adding a suitable gauge fixing term to the action,

$$\mathcal{L}_{gf} = \lambda \Phi, \tag{3.4}$$

where

$$\Phi = \frac{1}{\pi} \int d\sigma (e^{-2in_w \sigma} \partial_- X^2 - n_w R e^{-2in_w \tau}). \tag{3.5}$$

This allows us to solve the constraint $N_L - N_R = 0$ and determine $\alpha_{n_w}^2$. The parameters R and n_w will be the radius of the circle and the winding number of the string loop respectively. Also note that the gauge-fixing condition involves only the left-moving part of X^2 since $\partial_- X_R \equiv 0$.

The gauge-fixing condition and the constraint $N_L - N_R = 0$ can be solved (if $R \neq 0$) for $\alpha_{n_w}^2$ and $\alpha_{-n_w}^2$, respectively, as

$$\alpha_{n_w}^2 = \frac{n_w R}{\sqrt{2\alpha'}}, \tag{3.6}$$

$$\alpha_{-n_w}^2 = -\frac{\sqrt{2\alpha'}}{n_w R} \left(\sum_{m \neq n_w} \alpha_{-m}^2 \alpha_m^2 + \sum_{n \geq 1} \alpha_{-n}^j \alpha_n^j - \tilde{\alpha}_{-n}^i \tilde{\alpha}_n^i \right), \tag{3.7}$$

where $j = 2, \dots, D - 2$.

Upon quantization in this gauge, the mode decomposition of X_L^2 is different from the usual one in (3.2). It reads

$$\begin{aligned}
 X_L^2 = & \frac{1}{2}x^2 + \alpha' p^2(\tau - \sigma) - i\sqrt{\frac{\alpha'}{2}} \frac{1}{n_w} \alpha_{-n_w}^2 e^{2in_w(\tau - \sigma)} + \frac{i}{2} R e^{-2in_w(\tau - \sigma)} \\
 & + i\sqrt{\frac{\alpha'}{2}} \sum_{n \neq n_w} \frac{1}{n} \left(\alpha_n^2 e^{-2in(\tau - \sigma)} - \alpha_{-n}^2 e^{2in(\tau - \sigma)} \right), \quad (3.8)
 \end{aligned}$$

where $\alpha_{-n_w}^2$ should be interpreted as the rhs of (3.7). Therefore, the Fock space of states does not include $\alpha_{n_w}^2$ nor $\alpha_{-n_w}^2$ as operators acting on it. We also note that since the constraint $N_L = N_R$ is solved for states in this space, the mass formula is

$$\alpha' M^2 = 4N_R. \quad (3.9)$$

3.2 The state

Let us now consider a state, in the gauge of the previous subsection, of the form

$$|\phi\rangle = |\phi_w\rangle_L \otimes |N_R\rangle_R, \quad (3.10)$$

where the left-moving factor is a coherent state built on a left vacuum, $|0\rangle_L$,

$$|\phi_w\rangle_L = e^{-iRn_w(\alpha_{-n_w}^1 + \alpha_{n_w}^1)/\sqrt{2\alpha'}} |0\rangle_L, \quad (3.11)$$

and the right-moving part is a state of level N_R . Level-matching requires that

$$N_R = \frac{n_w^2 R^2}{\alpha'}. \quad (3.12)$$

Also, take vanishing center of mass parameters x^i and p^i . Notice that $\alpha_{n_w}^1 |\phi_w\rangle_L = -\frac{in_w R}{\sqrt{2\alpha'}} |\phi_w\rangle_L$. Using (3.7), this implies $\langle \phi_w | \alpha_{-n_w}^2 | \phi_w \rangle_L = n_w R / \sqrt{2\alpha'}$.

We can compute now the expectation value of the string coordinates in the normalized state $|\phi\rangle$, and find the circular loop of radius R we seek,

$$\begin{aligned}
 \langle X^1 \rangle &= i\sqrt{\frac{\alpha'}{2}} \langle \phi | -\frac{1}{n_w} \alpha_{-n_w}^1 e^{2in_w(\tau - \sigma)} + \frac{1}{n_w} \alpha_{n_w}^1 e^{-2in_w(\tau - \sigma)} | \phi \rangle, \\
 &= R \cos(2n_w(\tau - \sigma)), \quad (3.13)
 \end{aligned}$$

$$\begin{aligned}
 \langle X^2 \rangle &= i\sqrt{\frac{\alpha'}{2}} \langle \phi | -\frac{1}{n_w} \alpha_{-n_w}^2 e^{2in_w(\tau - \sigma)} + \frac{R}{\sqrt{2\alpha'}} e^{-2in_w(\tau - \sigma)} | \phi \rangle, \\
 &= R \sin(2n_w(\tau - \sigma)), \quad (3.14)
 \end{aligned}$$

in which the string winds the circle n_w times before closing in on itself, so that

$$\langle X^1 \left(\sigma + \frac{\pi}{n_w} \right) \rangle = \langle X^1(\sigma) \rangle, \quad \langle X^2 \left(\sigma + \frac{\pi}{n_w} \right) \rangle = \langle X^2(\sigma) \rangle. \quad (3.15)$$

There is no contribution from the right-moving bosonic excitations to the expectation value because we are considering that this sector is in an eigenstate of N_R . Since $N_R = N_L$

is solved, $|N_R\rangle_R$ is also an eigenstate of N_L . On the other hand, the coherent part $|\phi_w\rangle_L$ is annihilated by both N_R and N_L , and can be regarded as a ‘background’ on which we can put right-moving excitations. This left-moving factor is the only one that produces the circular loop of radius R in the X^1 - X^2 plane for a given mass. Once N_R , and hence the mass, is fixed, adding a left-moving component to this state would change the expectation value of the rhs of (3.7) and the shape of X^2 would change.

The mass and angular momentum on the state are

$$\langle\phi|\alpha' M^2|\phi\rangle = \langle\phi|4N_R|\phi\rangle = \frac{4n_w^2 R^2}{\alpha'}, \tag{3.16}$$

$$\langle\phi|J_{12}|\phi\rangle = \frac{n_w R^2}{\alpha'}, \tag{3.17}$$

where (3.17) is computed on the left-moving part of the state using

$$J_{12} = -\frac{i}{2} \sum_{n \neq 0} \frac{1}{n} \alpha_{-n}^{[1} \alpha_n^{2]}, \tag{3.18}$$

i.e., we are considering only the left-moving part because on average the contribution to the ensemble of right-movers of a given level has vanishing angular momentum. The result of dropping this restriction is considered below in section 3.4. Therefore we obtain the relations

$$J_{12} = \frac{n_w R^2}{\alpha'} = \frac{\alpha'}{4n_w} M^2. \tag{3.19}$$

These reproduce precisely the classical results (2.10) and (2.11).

Our construction of a state that semiclassically resembles a plasmid string has required a choice of gauge that is specific to the state we are considering, which fixes the radius of the string in the direction X^2 . This has allowed us to have non-vanishing values for $\langle X^1 \rangle$ and $\langle X^2 \rangle$ that coincide with the classical values in (2.7). However, one could still provide a different construction of the circular string while leaving unfixed the residual symmetry and imposing the constraint $N_L = N_R$ on physical states. This can be done in such a way that we obtain the required values of M , J_{12} , n_w , and with the root mean square position of the string being peaked on a circle of a given radius. Consider the state

$$|\psi\rangle = |\psi\rangle_L \otimes |N_R\rangle_R = \frac{1}{(J! 2^J)^{1/2}} (\alpha_{-n_w}^1 + i\alpha_{-n_w}^2)^J |0\rangle_L \otimes |N_R\rangle_R. \tag{3.20}$$

In this state we have

$$J_{12}|\psi\rangle_L = J|\psi\rangle_L, \tag{3.21}$$

$$N_L|\psi\rangle_L = n_w J|\psi\rangle_L \tag{3.22}$$

and $|N_R\rangle_R$ denotes an eigenstate of N_R . We must impose $N_R = n_w J$ in order to fulfill the level matching condition, $(N_R - N_L)|\psi\rangle = 0$. This implies

$$\alpha' M^2|\psi\rangle = 4n_w J|\psi\rangle, \tag{3.23}$$

so the classical relationship (2.11) is satisfied on the state, instead of only in expectation value like in (3.19). We know from [13] that any such state has $\langle X^i \rangle = 0$, but one can measure the size of the string loop by the operator,

$$r^2 = (X^1)^2 + (X^2)^2 = \alpha' \sum_{n=1}^{\infty} \frac{1}{n^2} \left(\alpha_{-n}^k \alpha_n^k + \tilde{\alpha}_{-n}^k \tilde{\alpha}_n^k \right) + \alpha' \sum_{n=1}^{\infty} \frac{1}{n}. \quad (3.24)$$

where $k = 1, 2$. The last term comes from reordering the operators to form the first sum, and gives a divergent contribution. This divergence was also noticed in [22], where, given that it is independent of the specific state, it was proposed that it be subtracted away. Doing the same, we find that our state has

$$\langle r^2 \rangle = \langle \psi | r^2 | \psi \rangle_L = \frac{\alpha' J}{n_w}. \quad (3.25)$$

If we identify $R = \sqrt{\langle r^2 \rangle}$ then we reproduce the classical relation (2.8). Hence this state has the values of J_{12} , M^2 and R that we are seeking. We could also describe the right-movers with a similar state, now in the 3-4 plane, thus yielding a non-zero J_{34} that reproduces the parameters of the classical state (2.7).

The states (3.10) and (3.20) share the main macroscopic parameters of the classical plasmid string, so both could be considered to provide a quantum microscopic description of it. However, we feel that the coherent state construction (3.10) captures more neatly through eqs. (3.14) the notion of a semiclassical rotating loop of string.

3.3 Degeneracies

The coherent left-moving part in (3.10) is chosen to yield a circular string with the required n_w and with a fixed radius R in the direction X^2 . Alternatively, we may say that we are fixing the mass and this radius. The right-sector component is instead only constrained to be at level N_R given by (3.12). This allows a large multiplicity for the string state. It is straightforward to compute it in the limit of high level N_R by studying the appropriate generating function with standard techniques [23].⁶

For bosonic strings ($D = 26$, $i = 1, \dots, 24$) we consider the partition function for right-moving states with arbitrary occupation number $\{N_n^i\}$ for each mode. The level of each of these states is $N_R = \sum_i \sum_{n=1}^{\infty} n N_n^i$, so the generating function for this system is

$$Z_B = \text{tr} e^{-\beta N_R} = \sum_{N_R=1}^{\infty} d_{N_R} z^{N_R}, \quad (3.26)$$

where $z = e^{-\beta}$ and d_{N_R} is the degeneracy of states at level N_R and the trace is taken over the state space. A standard saddle-point estimate yields

$$d_{N_R} \sim e^{4\pi\sqrt{N_R}}, \quad N_R \gg 1. \quad (3.27)$$

In fact, the generic result for all closed string theories (bosonic, IIA/B, heterotic) is

$$\log d_{N_R} \sim \sqrt{N_R}. \quad (3.28)$$

⁶Clearly we will obtain the same asymptotic degeneracy if instead of (3.10) we consider the state (3.20).

The precise numerical factor, which varies among the theories, will not be required for our purposes. Using (3.12) and (3.19) we find that, to leading order at large N_R , the entropy of our circular strings is

$$S \sim \sqrt{n_w J_{12}}. \tag{3.29}$$

3.4 Right-movers with angular momentum

We can also estimate the degeneracy of bosonic states when the right-movers have angular momentum in one direction, using the prescription and results of [24].

We add a term containing the angular momentum and a Lagrange multiplier ω to the Hamiltonian of the right-movers,

$$H_J = \sum_{n,i} n N_n^i + \omega J_{34}, \tag{3.30}$$

where we chose the non-vanishing component of the angular momentum on the desired plane to be

$$J_{34} = -\frac{i}{2} \sum_{n \neq 0} \frac{1}{n} \tilde{\alpha}_{-n}^{[3} \tilde{\alpha}_n^{4]}. \tag{3.31}$$

The partition function of interest is now

$$Z_J = \text{tr} e^{-\beta H_J}. \tag{3.32}$$

The saddle point approximation yields the degeneracy of states, which to leading order at large N_R and J_{34} is [24]

$$d_{N_R, J_{34}} \sim \exp \sqrt{4\pi(N_R - |J_{34}|)}. \tag{3.33}$$

For our states, the level is matched using (3.12) as before. The same result holds for all closed strings up to an overall numerical factor, so the leading-order estimate for the entropy is

$$S \sim \sqrt{\frac{n_w^2 R^2}{\alpha'} - |J_{34}|} = \sqrt{n_w J_{12} - |J_{34}|}. \tag{3.34}$$

It is worth noting that at each level the number of states with nonvanishing J_{34} is a subleading fraction, proportional to $1/\sqrt{N_R}$, of those with $J_{34} = 0$.

4. Supergravity solution for a rotating loop of fundamental string

We now want to find a solution that describes the supergravity fields sourced by the rotating loop of fundamental string of the previous sections. We first discuss the five-dimensional ring (so the additional five space dimensions are assumed to be compactified, without any non-trivial physics arising from the compactification), since in this case exact explicit solutions are available. Afterwards we describe the extension for arbitrary $D \geq 5$, where approximate solutions can also be constructed.

4.1 The 5D solution and its physical parameters

The solutions we seek must describe a non-supersymmetric ring-like object, with non-vanishing angular momentum, and possessing a dipole of the Kalb-Ramond field $H_{(3)}$, but no other gauge charges. The wiggly structure of the oscillations is assumed to be too small to be resolved by the supergravity fields, and the degeneracy of the solutions is not sufficient to give rise to a regular horizon, at least at the level of the two-derivative supergravity action. So we expect a singularity on the ring.

Supergravity solutions with these properties were actually presented in [14]. In order to establish precisely the identification we need to show that the relations (3.19) and (3.29) are also satisfied by the supergravity solutions, at least in the regime in which the string coupling constant is small. To this effect, we need to extract the physical parameters of the solution. In the string frame, it takes the form⁷

$$ds^2 = -\frac{(1+\mu y)(1-\mu x)}{(1-\mu y)(1+\mu x)} \left(dt + R \mu \sqrt{\frac{1+\mu}{1-\mu}} \frac{1+y}{1+\mu y} d\psi \right)^2 + \frac{R^2}{(x-y)^2} (1-\mu^2 x^2) \left[\frac{y^2-1}{1-\mu^2 y^2} d\psi^2 + \frac{dy^2}{y^2-1} + \frac{dx^2}{1-x^2} + \frac{1-x^2}{1-\mu^2 x^2} d\phi^2 \right], \quad (4.1)$$

with Kalb-Ramond potential

$$B_{t\psi} = -R \mu \sqrt{\frac{1-\mu}{1+\mu}} \frac{1+y}{1-\mu y}, \quad (4.2)$$

and dilaton

$$e^{2\phi} = \frac{1-\mu x}{1-\mu y}. \quad (4.3)$$

Readers unfamiliar with the set of coordinates employed here may find the explanation in [15] helpful. Briefly, $-R/y$, with $y \in (-\infty, -1]$, is a sort of radial coordinate in the direction away from the ring, and x and ϕ parametrize two-spheres that link the ring once, with $x \sim \cos \theta$ and $x \in [-1, 1]$. Regularity at the axes of ϕ and ψ rotations requires the periodicities $\Delta\phi = \Delta\psi = 2\pi\sqrt{1-\mu^2}$. The solution has a (timelike) naked singularity at $y = -\infty$, where the ring lies, and an ergosurface around it, with topology $S^1 \times S^2$, at $y = -1/\mu$. The ring rotates along the ψ direction.

There are two parameters, R and μ : R corresponds to the ring radius and sets the scale for the solution. The dimensionless μ is related to the dipole of the $B_{(2)}$ field. This is proportional to the winding number of the string obtained by integrating the flux of $H_{(3)} = dB_{(2)}$ across a 2-sphere that links the string once (i.e., any sphere at constant $y \in (-\infty, -1)$),

$$n_w = \frac{\alpha'}{8G} \int_{S^2} e^{-2\sqrt{\frac{2}{3}}\phi} * H_{(3)} = \frac{\pi\alpha'}{2G} R\mu. \quad (4.4)$$

⁷This is obtained from the solutions in [14] by setting $\nu = 0$ (for extremality), $N = 1$ (for the dilaton coupling of fundamental strings), $\lambda = \mu$ (for equilibrium), and finally changing to string frame.

We can use this equation to eliminate μ in favor of quantities with direct physical meaning. In terms of these, the (Einstein frame) ADM mass and angular momentum of the solution are

$$M = \frac{2R}{\alpha'} n_w \left(1 + \frac{G n_w}{\pi \alpha' R} \right), \quad (4.5)$$

$$J_{12} = \frac{R^2}{\alpha'} n_w \left(1 + \frac{2G n_w}{\pi \alpha' R} \right)^2. \quad (4.6)$$

We are denoting the plane where the ring rotates as the 1-2 plane, to make contact with previous sections.

The presence of Newton's constant G in these expressions is a sign of the effect of self-gravitational interaction within the ring. These effects are not included in the construction of the semiclassical string state, so in order to make the comparison we must neglect them here too. This requires that we consider 'thin rings', such that the 'charge radius' μR in directions transverse to the ring is much smaller than the ring radius,

$$\frac{G n_w}{\alpha'} \ll R. \quad (4.7)$$

In effect, we linearize the solution around flat space. Then, to zero-th order in $G n_w / \alpha' R$ we have

$$M = \frac{2R}{\alpha'} n_w + O(G), \quad J_{12} = \frac{R^2}{\alpha'} n_w + O(G). \quad (4.8)$$

To this order, these reproduce exactly (3.19). The supergravity solution thus possesses the correct physical parameters to match the semiclassical string loop.

Let us analyze briefly the effect of the $O(G)$ corrections. They appear when we account for the fact that the ring attracts itself through interactions mediated by gravitons as well as by $H_{(3)}$ and ϕ exchange. In the thin ring regime (4.7) the interaction is at large distance so these massless fields give rise to Newtonian and Coulombian forces. If we expand (4.5) and (4.6) beyond leading order, we find

$$M = 2\sqrt{\frac{n_w J_{12}}{\alpha'}} - \frac{G}{2\pi} \frac{M^2}{R^2} + O(G^2). \quad (4.9)$$

The correction to the mass at first order in G has indeed the form of a Newtonian potential energy in five dimensions — in fact an attractive one, as it should be since not only gravity but also the other fields have a self-attractive effect on the ring.

Note that these corrections, which arise from closed string interactions are *classical*. There will be other terms at the same order in the string coupling with an interpretation as quantum corrections. They will in fact give a decay rate for the state, to be discussed below.

4.2 $D \geq 6$

In $D \geq 6$ we do not have explicit exact solutions for dipole rings, but the methods of [25] allow to construct approximate solutions in the regime we are interested in, namely, to first

order in the parameter

$$\frac{Gn_w}{\alpha'R^{D-4}} \ll 1. \tag{4.10}$$

The method involves solving the supergravity equations in two regions, first at distances $r \gg (Gn_w/\alpha')^{\frac{1}{D-4}}$, where the linearized approximation around flat space is valid, and then near the ring core, $r \ll R$, where we perturb around the limit of a straight ($R \rightarrow \infty$) fundamental string with momentum. If (4.10) holds, then there is an ample region where the two approximations are simultaneously valid and the respective solutions can be matched.

The construction is straightforward, if a little tedious, but we do not need to develop it in full in order to derive the result we seek, namely the relations (3.19) —which, observe, are independent of the number of dimensions. We begin by considering the solution near the ring to zero-th order in the parameter (4.10). This is simply the solution for an extremal fundamental string with momentum (FP-string), which in string frame is [3, 4]

$$ds^2 = h^{-1} \left(-dt^2 + dz^2 + \frac{p}{r^{D-4}}(dt - dz)^2 \right) + dr^2 + r^2 d\Omega_{D-3}^2 \tag{4.11}$$

with

$$h = 1 + \frac{q}{r^{D-4}}. \tag{4.12}$$

The winding number is given by the charge q as

$$n_w = \frac{(D-4)\Omega_{D-3}\alpha'}{8G}q, \tag{4.13}$$

and if the direction z is periodically identified $z \sim z + 2\pi R$, then the momentum parameter p is quantized as

$$\frac{n_p}{R} = \frac{(D-4)\Omega_{D-3}R}{8G}p. \tag{4.14}$$

At large distances from the ring, $r \gg q^{1/(D-4)}, p^{1/(D-4)}$, the gravitational field is the same as that created by a distributional energy-momentum (measured in Einstein frame)⁸

$$\begin{aligned} T_{tt} &= \frac{D-4}{16\pi G} (p+q) \delta^{(D-2)}(r), \\ T_{tz} &= \frac{D-4}{16\pi G} p \delta^{(D-2)}(r), \\ T_{zz} &= \frac{D-4}{16\pi G} (p-q) \delta^{(D-2)}(r), \end{aligned} \tag{4.15}$$

plus a linear string source $\propto q$ for $H_{(3)}$ that we need not specify here. We now consider this same distributional source, but lying along a ring of radius R in D -dimensional flat spacetime, parametrized by an angle $\psi = z/R \in [0, 2\pi)$. Then the momentum p along z becomes proportional to an angular momentum J_{12} along ψ , and the charge q becomes

⁸We normalize $\int_{B^{D-2}} \delta^{(D-2)}(r) = \Omega_{D-3}$, where B^{D-2} is a ball that intersects the string once.

the dipole of the ring. The linearized supergravity equations can be solved for this source, providing a solution valid at distances $\gg (Gn_w/\alpha')^{\frac{1}{D-4}}$. It is not difficult to do this, but in order to extract the physical parameters of the ring we need only notice that at any finite R the ring will be in mechanical equilibrium only when its tendency to collapse under its tension is balanced by the centrifugal repulsion. The condition for this to happen is that

$$\frac{T_{zz}}{R} = 0. \tag{4.16}$$

This was argued in [25] from several points of view: it follows from the classical equation of motion for a probe brane, derived in [26] as a consequence of conservation of the stress-energy tensor. Perhaps more appropriately for our present purposes, ref. [25] showed that it follows from the requirement of absence of singularities on the plane of the self-gravitating thin ring, away from the ring location. Both arguments lead to (4.16) in the present case. Imposed on (4.15) it implies that the ring will be in mechanical equilibrium only when

$$p = q. \tag{4.17}$$

Using (4.13) and (4.14), which remain valid to this order of approximation, this gives the mass and angular momentum of the ring as

$$M = 2\pi R \int_{B^{D-2}} T_{tt} = \frac{(D-4)\Omega_{D-3}}{4G} qR = \frac{2R}{\alpha'} n_w, \tag{4.18}$$

$$J_{12} = 2\pi R^2 \int_{B^{D-2}} T_{tz} = n_p = \frac{R^2}{\alpha'} n_w. \tag{4.19}$$

So we find that the expressions (3.19) are again exactly reproduced. Hence thin extremal dipole rings provide the correct supergravity description of semiclassical circular loops of string in any $D \geq 5$. It may be worth noting that the integrated expressions for T_{tt} and T_{zz} correspond to the energy and effective tension introduced in (2.22) and (2.23), and so (4.17) is clearly the same as (2.24).

Arguments of the sort discussed in the previous subsection indicate that the first $O(G)$ corrections away from the thin ring limit will reduce the mass by an amount $\propto GM^2/R^{D-3}$.

Finally, note that the supergravity solution for a fundamental string with only fermionic excitations should take the same form as (4.11), so our construction also applies to the circular strings of [12].

4.3 Entropy

The geometry (4.1) does not have a horizon, but instead a naked singularity, so it would seem that there is no entropy associated to it. How can we then identify it with the perturbative string state, which has a degeneracy (3.29)? The resolution is that the entropy (3.29) is too small to show up as a macroscopic entropy in the leading low-energy effective supergravity description [5]. The situation is in fact closely analogous to the ‘small black holes’ that correspond to elementary string states wrapped on a circle. In recent years it has been argued that the inclusion of higher-derivative corrections to the supergravity action removes the singularity at the core and replaces it with a ‘stretched’ horizon. The Wald

entropy of this horizon precisely reproduces the microscopic entropy of the fundamental string state [6].

It turns out that we can immediately apply to our rotating rings the same arguments and results that [5, 27] developed for the straight fundamental string. The reason is that in the thin ring limit (4.7) the near-horizon geometry is exactly the same as that of the straight string, namely eq. (4.11), now with $h = q/r^{D-4}$. Deviations from this near-horizon geometry come from self-interaction of the loop, which we are neglecting. Ref. [27] developed a scaling argument, based only on this near-horizon geometry, to the effect that the entropy of the stretched horizon is

$$S \sim \sqrt{n_w n_p}, \tag{4.20}$$

where n_p are the units of momentum along the string. This is valid in any dimension $D \geq 5$. When we bend the string to form a circle, we have $n_p \rightarrow J_{12}$, so

$$S \sim \sqrt{n_w J_{12}}, \tag{4.21}$$

which reproduces the parametric dependence (3.29) of the microscopic string state. Fixing the precise factor requires control over higher-derivative corrections, which is currently unavailable for generic D . However, the scaling (4.21) is a robust result.

4.4 Second angular momentum: tubular loops

We have seen that it is possible for the right-movers on the string to contribute a second angular momentum J_{34} in a plane orthogonal to the plane of the loop. This angular momentum is bounded above by (2.21). It is now natural to ask what is the supergravity counterpart of these solutions.

There do exist some exact solutions for black rings with two independent angular momenta [28–30], and they all satisfy a bound $|J_{34}| < J_{12}$. None of these, however, is a black ring with a dipole of $H_{(3)}$ and no other gauge charge. We will argue that, in fact, the supergravity solution we seek here is not a ring but instead a torus — i.e., not a loop of string but a loop of tube, or *tubular loop*.

Begin by considering the solutions in the limit that the radius of the loop $R \rightarrow \infty$. In this limit the solutions are BPS, so they should correspond to supersymmetric rotating FP strings. Such solutions are well-known to be helical strings that in the supergravity description are smeared along the direction of the axis and thus describe a tube $S^1 \times \mathbf{R}$ — this is the topology of the locus where the spatial section of the supertube worldvolume lies. The exact solutions have been presented in [21]. We now want to bend the tube axis into a circle of radius R , so the result will be a toroidal $S^1 \times S^1$ tube, see figure 2.⁹

When R is large, an approximate supergravity solution (which breaks all supersymmetries) can be constructed, in any $D \geq 5$, using the methods of [25]. Just like in section 4.2,

⁹The helical strings of [21] are U-dual to the supergravity supertubes of [31]. However, T-duality along the loop is not a valid symmetry among tubular loops (it has fixed points at the axis). Since the D0-F1/D2 supertube of [32, 31] does not carry momentum along the tube direction, it will not, if bent to form a loop, possess the centrifugal motion to balance its tension.

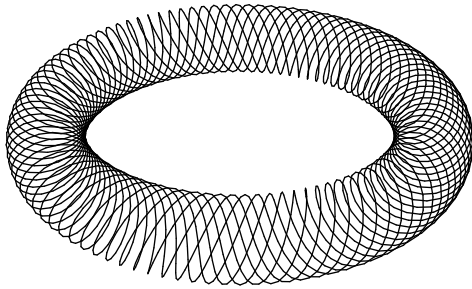


Figure 2: Tubular loop: when the radius of the helix oscillations is macroscopically large, instead of a loop of string we obtain a toroidal tube (for large n_w the string is very uniformly spread on the tube; in the picture $n_w = 7$). It rotates in the two circles of the torus, with $|J_{34}| \leq n_w J_{12}$.

we do not actually need to perform this construction explicitly in order to extract the physical parameters in the limit where the system is weakly interacting with itself. It has been shown that a supergravity supertube rotating in the 3-4 plane satisfies the bound [21, 31]

$$|J_{34}| \leq n_w n_p. \quad (4.22)$$

When we bend the tube on the plane 1-2 to form the torus we identify $n_p = J_{12}$. Moreover, it is easily seen that mechanical equilibrium of the tube, i.e., the condition that $T_{zz} = 0$, requires again that (2.24) holds. Thus we recover (2.20) and (2.21).

The string state has a microscopic degeneracy given by (3.34), whose origin is the possibility of a wiggly structure that ‘thickens’ the tube. In turn, the supergravity solution for the supertube develops a stretched horizon with an entropy that, in any $D \geq 5$, is

$$S \sim \sqrt{n_w n_p - |J_{34}|}. \quad (4.23)$$

This has been derived in [33] using the scaling arguments of [27]. In the thin tube limit we can apply this formula to our tubular loop, with the substitution $n_p \rightarrow J_{12}$. Then the entropy (4.23)

$$S \sim \sqrt{n_w J_{12} - |J_{34}|} \quad (4.24)$$

reproduces (3.34). Observe that the topology of the stretched horizon is in this case $S^1 \times S^1 \times S^{D-4}$. Angular momenta on other planes would blow up new circles.

4.5 String decay by superradiant emission

The quantum state of the fundamental string (3.10), (3.11), contains excitations in both left and right sectors, so it is not a BPS state and should have a finite lifetime when interactions are switched on.

On the other hand, the supergravity solution for the ring (4.1) is also expected to decay. The decay is unlikely to happen at the classical level, since this ring is not affected by any of the black ring instabilities discussed in [34]. But quantum decay is certainly expected:

not by Hawking emission, since the solution is extremal and has zero temperature,¹⁰ so it does not emit thermal radiation. But it has an ergoregion. Extremal black holes with an ergoregion surrounding a horizon with angular velocity Ω_H , do emit spontaneously modes $\Psi \sim e^{-i(\omega t - m\phi)}$ that satisfy a superradiant bound on their frequency $\omega < m\Omega_H$ [35]. This emission carries away some of the angular momentum of the black hole. The calculation of this process in the background of the ring (4.1) is technically complicated, since the variables x and y cannot be separated in the wave equation.¹¹ Nevertheless, on general grounds we expect that this ring, as well as its $D > 5$ counterparts, will decay by such superradiant emission.

It is possible to identify the counterpart of this radiation in the microscopic model. The main ideas are contained in the microscopic picture of black hole superradiance developed in [16, 17] using the microscopic dual of an extremal rotating black hole with an ergosphere. This system possesses essentially the same features as our configurations. In both cases we have a state of a 1+1 CFT where the left-moving sector is filled with coherently polarized excitations so it accounts for the angular rotation of the horizon and is at zero temperature. The right-moving sector, instead, is in a thermal ensemble and accounts for the entropy, and possibly for angular momentum in an orthogonal plane. The state can decay by an interaction between the two sectors, which results in the emission of a massless closed string. The left-moving excitation provides angular momentum, so the emitted quantum will carry away some spin from the system. In our case, the emission will reduce the value of R , so the radius of the loop, and the angular momentum J_{12} , will decrease. It is natural to expect that this decay corresponds to the superradiant emission of the black ring.

For the extremal rotating black hole in [17], it is possible to show that the microscopic model implies that the emitted quanta do indeed satisfy the bound $\omega < m\Omega_H$. It should be interesting to also match, at least parametrically, the superradiant frequency bound for our circular strings using their micro and macro descriptions. Note that the decay of a large semiclassical state is expected to be slow, so these circular strings should be long-lived. In addition, as a result of the decay, the left sector of the string will gradually lose coherence. In the supergravity side, the temperature will be raised from zero and the black ring will become non-extremal.

Finally, notice also that the classical plasmid string of section 2, or a generic state with arbitrary wiggly profile, has a varying mass-quadrupole moment so when it is coupled to gravity it is expected to radiate gravitational waves.¹² However, this radiation is strongly suppressed for large N , i.e., when the wiggle amplitude is small and rotational invariance along the loop is approximately recovered. We expect that the superradiant emission in the supergravity solution (which is rotationally invariant) can be set in correspondence with this suppressed classical radiation.

¹⁰This is properly defined after the near-horizon geometry is regularized with higher-derivative corrections to an AdS type of horizon.

¹¹In principle the horizon needs to be regularized by higher-derivative corrections, but presumably a simple absorptive boundary condition at the stretched horizon is enough to derive superradiance.

¹²The decay of the fundamental string through emission of classical gravitational waves is considered in [36].

5. Discussion

The main thread of the paper has been the idea that black rings can be regarded as circular strings. This is a different perspective than viewing them as supertubes dimensionally reduced along the tube direction, as first proposed in [37]. In the context of states of the fundamental string, the latter view is taken in [38, 33], where certain BPS states of the string are related to a class of small black rings. These rings are the helical strings, smeared along the helix axis, of [21], which are U-dual to supergravity supertubes [31]. Dimensional reduction along the axis direction yields a two-charge ring that saturates a BPS bound. The configurations we have discussed are different, and in some respects simpler, than these. They are not supersymmetric since they only have dipoles, not conserved charges, hence they are more similar to neutral black holes. Even when we have discussed supertubes, we have broken their supersymmetry to form a tubular loop, not a ring.

The limitation to thin rings in our correspondence between the different descriptions of the circular string may look like an important deficiency compared to what can be achieved for supersymmetric black rings. However, we believe that the understanding we have obtained of the properties of black rings, when viewed as circular strings, is significant. In particular, we have in mind a main drawback of the otherwise successful identification of the microstates of supersymmetric black rings [28] when regarded as circular versions of the MSW string [39, 40]. This model does not appear to account for the fact that the ring wraps a contractible cycle, and that therefore its radius is fixed in terms of parameters such as the angular momentum and mass. Our analysis of dipole rings does precisely this, and may provide hints about what is missing in the supersymmetric ring description of [39, 40]. Besides, our picture may form the basis for a better understanding of the microphysics of neutral black rings, following the suggestions in [14].

Our discussion of the regularization of the stretched horizon has been based on the expectation that higher-derivative corrections will reveal a small AdS-type horizon [6, 27]. However, there is another perspective on this issue, suggested in [41] as a prototype for the ‘fuzzball’ proposal: the FP string solution (4.11) is only an effective, coarse-grained geometry for more fundamental solutions that are in one-to-one correspondence to individual horizonless string states. Such supergravity microstates are known explicitly for the straight string (4.11), and generically describe a wiggly pattern of oscillating waves along the string. In this spirit, we should find geometries for each of the classical string configurations in section 2. Solving exactly the supergravity equations to find these configurations is too difficult, but one might try the method of matched asymptotic expansions of [25] to build an approximate solution in the thin ring regime (4.10). Presumably this is still technically challenging, but we can anticipate an important feature: since the wiggles break the exact rotational invariance of the ring, they will give rise to a varying quadrupole and the configuration will radiate gravitational waves — just like we argued at the end of section 4 for the classical plasmid string. These waves will carry not only mass but also angular momentum away from the ring. However, the emission will be suppressed for small wiggles. The coarse-grained geometry (4.1) is rotationally invariant and does not emit classical radiation, but it decays through superradiant emission, so, again, it is

natural to conjecture that both decays are dual descriptions of the same phenomenon. In a very recent paper [42], a closely similar emission from a set of microscopic states is also put in correspondence with macroscopic superradiant decay.

Finally, our arguments and constructions have all referred to $D \geq 5$, since the four-dimensional case presents peculiarities of its own. First, the semiclassical construction of section 2 in terms of purely bosonic wiggly excitations would require that the string oscillates as well in at least one of the 1-2 directions, and this typically leads to cusps. Still, this problem is avoided if the excitations are purely fermionic, see [12]. The approximate supergravity construction described in section 4 seems to extend as well to $D = 4$, in spite of the logarithmic behavior of the solution near the string core. However, in $D = 4$ the scaling argument of [27] fails to produce a regular horizon of size parametrically larger than the string length [43], essentially because q and p are dimensionless and there is no scale for the horizon radius other than the string length. The conclusion seems to be that four-dimensional loops of string are possible, but they give rise to supergravity rings with string-scale cores and not (small) black rings. It may be interesting to further investigate this system.

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